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Memorandum to: J. R. Harbison
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Chairman, Research Committee

Subject: Research Report, No. 318; "Bridges: Synthesis of Load Histories and Analysis of Fatigue;" KYP-69-11; HPR-1(7), Part III

We have been delayed in fulfilling some previous commitments and goals related to the evaluation of life-expectancy of certain major bridges -- specifically, the more aged bridges over the Ohio River. Concepts and methodologies are, indeed, enabling prerequisites. The report, here submitted, presents a rational approach to the "soft" synthesis of "used up" and "remaining" fatigue life of a bridge. The methodology remains incomplete: it does not, at present, provide a completely-computerized scheme. Such a program seems feasible. The most difficult expressions would be those "transforming load distributions into stress distributions." The parameters needed are specific for each element or member of the structure. We have performed the complete computations using assumed or hypothetical load-stress parameters.

While we have been pursuing the synthesis approach, a promising development in technology and instrumentation has emerged. It has become possible to place a small listening probe on a steel member and to record or count internal dislocations or cleavages accompanying stressings. It also seems feasible to assess the state or degree of fatigue damage accumulated -- that is, from the noise rate. We have prepared a proposal, now pending, to undertake a study (KYHPR-72-70) of real, so-called critical members of selected bridges. This approach, together with or independent of the synthesis, may lead to fatigue testing of coupons -- this being considered the final resort.

Respectfully yours,

Jas. H. Havens

Jas. H. Havens
Director of Research

JHH:dw
Attachment
cc's: Research Committee

Research Report

318

BRIDGES: SYNTHESIS OF LOAD HISTORIES AND ANALYSIS OF FATIGUE

KYP-69-11; HPR-1(7), Part III

by

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INTRODUCTION

Repeated stressing of metals above certain limits induces inter- and intra-crystalline dislocations and cleavages and eventually cracks which propagate to failure. Some authorities consider crack propagation to be a separate and discrete stage in the failure process. The internal damage is insidiously cumulative and irreversible. This phenomenon was recognized as early as 1829 and was termed fatigue as early as 1839*(1)*. From the beginning of fatigue testing (Wohler, 1858-1870), results have been reported as S-N, S-log N, or log S-log N curves, where N is the number of repetitions of stress S. From a structural design point of view, the purpose of fatigue testing then was to find the endurance or fatigue limit (i.e., f_{EL}) and so to establish the design or working stress (for many steels, f_{EL} came to be regarded as 55 percent of f_y, the yield stress, or 46 percent of f_u, the ultimate strength).

In order to plot S-N graphs, it was necessary to test many specimens at several stress levels -- each in simple, repetitive cycling. About 1910, compound loading tests evolved. The linear summation of cycle ratios is believed to have originated with A. Palmgren in 1924. In this country, it was proposed by B. F. Langer in 1937, although credit is often given to M. A. Miner (1945). This hypothesis merely suggested that the fractional damage in a specimen caused by N repetitions of a stress is the ratio of the number of those repetitions to the number of repetitions at the same stress level which would cause failure if continued (determined from other specimens). It is inherent in this notion that fractional damages add and that the totality of fractions equals one. It is therefore possible, on this premise, to predict remaining fatigue life from S-N envelopes and to do so in terms of compound stressings. Unfortunately, the simplicity implied here is perhaps unreal. Linearization of the S-N curves is a prerequisite. Indeed, the variability attending fatigue tests superimpose. If the "mean" or "median" fatigue life is used or sought, it should be so identified; however, if probability of failure is to be considered, other statistical parameters must be brought to bear. Incertitudes otherwise limit the summation of damage increments (fractions) to a value less than 1 -- perhaps 0.80. Some commentaries have suggested fail-safe values of 0.30. Adjustments necessary to achieve similitude with real-life situations would yet seem admissible. More complete reviews of fatigue technology are available elsewhere *(2, 3, 4)*.

Whereas many bridges built more than 50 years ago carry today's traffic and were then designed to be immune to fatigue with respect to the standard load (once a 15-ton road roller) and whereas legal allowable gross weights of trucks have increased more than fourfold in less than 18 years (Table 1), the possibilities of fatigue failure becoming imminent was deemed somewhat demanding of investigation and analysis. The recent catastrophic failure of the bridge at Point Pleasant, West Virginia *(5)* and the necessary subsequent retirement of the C & O bridge at Covington (US 25) *(6)* are conspicuous but contrasting events in engineering history. Each, in its respective way, is an example of delimited service life.

TABLE 1.

WEIGHT RESTRICTIONS ON KENTUCKY HIGHWAYS (1918 - 1969)[1]

YEARS	MAXIMUM ALLOWABLE GROSS WEIGHT (TRUCKS AND TRUCK COMBINATIONS) (pounds)	MAXIMUM ALLOWABLE SINGLE-AXLE WEIGHT (pounds)	MAXIMUM ALLOWABLE TANDEM-AXLE WEIGHT (pounds)	MAXIMUM ALLOWABLE GROSS WEIGHT (VEHICLES OTHER THAN TRUCKS) (pounds)
1918-1936				30,000
1936-1942	18,000			30,000
1942-1946	18,000 28,000[2]	16,000[2]		30,000
1946-1956	42,000	18,000		42,000
1956-1960	59,640	18,000		59,640
1960-	73,280	18,000	32,000	73,280

[1] Based on information obtained from Kentucky Revised Statutes up to June 13, 1968.

[2] National Emergency Highways only.

The purpose of this study was to reconstitute or synthesize the load history of certain bridges and to develop a logical method of estimating their remaining service life. Presumably, coupons or specimens of steels could be removed from the bridges and subjected to fatigue tests. Perhaps this will become an eventual recourse -- or perhaps accoustical emission evaluations *(7)* will suffice in the near future. Nevertheless, the association of load histories with fatigue damage is a necessary, independent phase of analysis in the

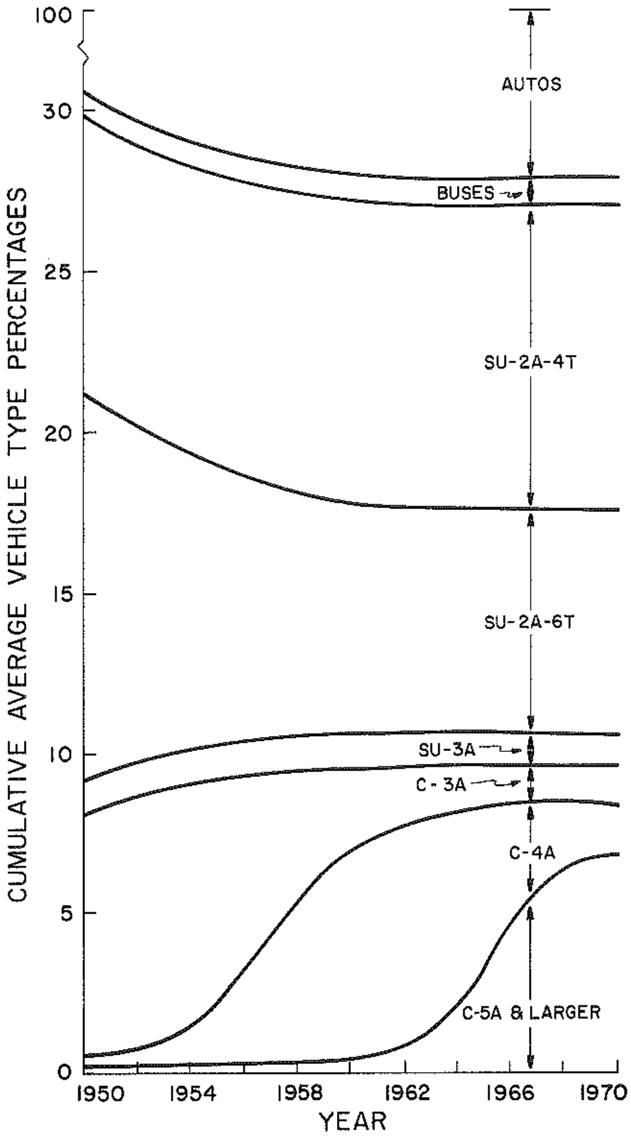

Figure 1. Mean Vehicle Type Percentages on Rural Highways

Equation 3 can be modified to give the probability that these vehicles will pass the point of interest within a specified time interval "t":

$$P_{niG} = P_i^n \, P_{G(t)}, \qquad 4$$

where $P_{G(t)}$ is the probability of a gap being of average length G(t). The gap length probabilities required in Equation 4 have been developed previously for specific bridges spanning the Ohio River from Kentucky *(11)*. Final probability curves were developed by recording actual vehicle gap lengths (in seconds) and then converting the gap distributions from units of time to units of length by considering the average vehicle spot speeds at these locations.

Assuming that the gap distances are equal, the average gap length for vehicles within the critical length of roadway (L) is found to be

$$G(t) = (L - n_i VL_i)/(n_i - 1/2), \qquad 5$$

where VL_i is the average length of vehicle type "i" (see Table 2). The average gap for mixed traffic (see Figure 2) in one direction is found from

$$G_{mix} = [L - \sum_{\text{all } i} n_i \, VL_i] \; / \sum_{\text{all } i} (n_i - 1/2) \qquad 6$$

where $\sum_{\text{all } i} n_i \, VL_i < L$.

TABLE 2.

AVERAGE VEHICLE LENGTHS USED IN FATIGUE ANALYSIS

VEHICLE TYPE	AVERAGE VEHICLE LENGTH[1] (feet)
Auto	19 [2]
SU-2A-4T	21
SU-2A-6T	24
SU-3A	28
C-3A	45
C-4A	48
C-5A	48
C-6A	52
Auto	20
Single-Unit	25
Combination	47

[1]Average vehicle length for Bridges 1 - 13 from Reference 8.
[2]AASHO *(9)*.

Because of the large number of variable combinations, it became necessary to restrict the vehicle classification to the three vehicle types given in Table 3. Considering the vehicle classification system shown in Table 3, the probability of any one-directional, mixed is found to be

$$P_D = ADT \times D/\ 255{,}640\ SP\ , \qquad 8$$

where SP is the average spot speed (in miles per hour) at the point of interest.

The traffic composition probabilities for "r" lanes of a one-directional highway can be found from

$$PR = [\prod_{\text{all } r} (P_D P_{n_i,n_2,n_3,r})]/P_D\ . \qquad 9$$

Corresponding probabilities of two-lane, two-directional traffic can then be computed from

$$P = P_D \prod_{r=1}^{r=2} P_{n_1,n_2,n_3,r}\ . \qquad 10$$

Although the above probabilities are based on numerous assumptions, the fact that traffic operation is continuous requires such assumptions. Any such probability derivation must be made with similar qualitative assumptions, although the quantitized criteria are subject to re-evaluation based on actual traffic and loading studies at the particular point under consideration. Here, the number of vehicle loading combinations to be considered by the above probability equations increases rapidly as the length of roadway under study increases.

Use of Probability Equations

Prior to the development of the final loading distributions, traffic data must be analyzed to find the frequency of occurrence of each vehicle grouping. Based on these frequencies (probabilities), the total number of repetitions for a particular vehicle grouping during an analysis period of "Y" years can be computed from

$$N_{n_1,\ n_2,\ n_3} = 365 \sum_{\text{all years}} ADT \times \bar{P}. \qquad 11$$

The total number of vehicle groups to be analyzed by Equation 11 during the minimum time period for an r-lane highway is obtained from

$$TOT = [\prod_{\text{all } i} (MN_i + 1)]^r, \qquad 12$$

where $MN_i = L/VL_i$ = maximum number of vehicles of type "i" that can occur in length L at one time. It should be noted that TOT represents the maximum number of vehicle groupings to be analyzed. Vehicle groupings can be analyzed for fatigue contributions determined from a potential maximum to a potential minimum. Once the stress level falls below the endurance limit of the member being analyzed, the computational routine presented in Equation 12 is terminated.

Gross Load Distribution

Associated with each loading configuration is the probability distribution of the gross weight of that particular loading condition. The derivation of such a probability requires a knowledge of the parameters obtained from the previous sections:

1. The total number of repetitions of each possible loading configuration ($N_{n_1,\ n_2,\ n_3}$) during each year.
2. The probability ($P_{n_1,\ n_2,\ n_3}$) of the occurrence of each loading condition in the length under consideration for each year.
3. The individual gross vehicle load probability distribution (PL_i) for each vehicle type considered in the fatigue analysis.

The basic procedure considers all possible loading combinations for each gross vehicle load interval of GL_i for each vehicle of each vehicle type found in the vehicle loading configuration under consideration. The total gross loading probability distribution having q intervals can be found by combining the individual gross bridge loading distributions corresponding to the individual loading configurations ($P_{GL_{ijq}}$) by the following:

$$P_{TL_q} = \sum_{\text{all LC}} P_{GL_{ijq}} P_{n_i,n_2,n_3}\ . \qquad 13$$

The P_{GL} terms can be developed for a particular loading distribution from

$$\begin{aligned} P_{GL_{ijq}} = {} & P_{GL_{1,1,q}} \times P_{GL_{1,2,q}} \times \dots P_{GL_{1,N_1,q}} \times \\ & P_{GL_{2,1,q}} \times P_{GL_{2,2,q}} \times \dots P_{GL_{2,N_2,q}} \times \\ & P_{GL_{3,1,q}} \times P_{GL_{3,2,q}} \times \dots P_{GL_{3,N_3,q}} \end{aligned} \qquad 14$$

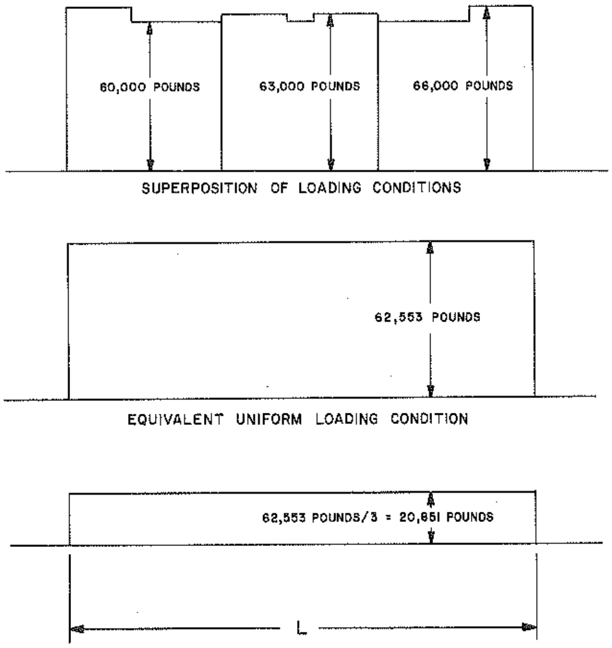

Figure 5. Equivalent Uniform Loading Condition

equivalent uniform loading for the design vehicle positioning and configuration is done by

$$LC_E = f(LC) \ , \tag{16}$$

where f(LC) is the load equivalency function relating these loads. Application of these modifications to the individual load distributions $P_{GL_{ijq}}$ allows the determination of the final equivalent loading distribution for input into the analysis presented in the next section.

It seems logical that a generalized function f(LC) for transforming displaced loadings to equivalent point loadings could be developed. Specified loads can be simulated at different points on the span of a particular bridge. The stress induced in the critical member by the load placed at each of these positions could be computed. The magnitude of the loads at the critical point of the span corresponding to these stresses could then be computed. Knowing this, the ratios of the equivalent loads at the critical point to the load at different positions can then be determined. A plot of such points -- load ratio versus position of load in the critical length -- is then made (see Figure 6). A line of best fit is then obtained either statistically or visually. This curve is the desired function f(LC). The determination of this curve for numerous members of the same bridge and for a number of bridges should provide the necessary data required for developing a generalized relationship for f(LC).

Such a complex equivalent load analysis, as described above, did not seem to be reasonable at this time. An attempt was made to develop the fatigue analysis methodology so that quantitized results from any equivalent load analysis as described earlier in this section could be easily inserted into the methodology.

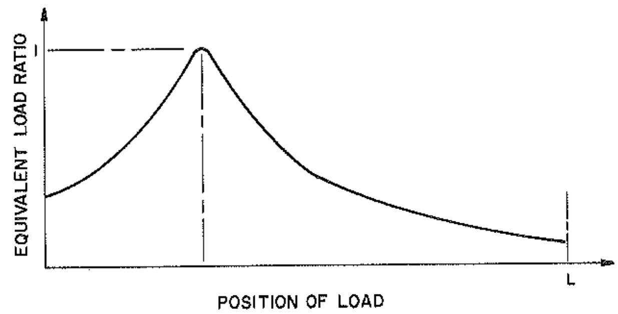

Figure 6. Example of Effect of Placement of Load on Force Transmitted to Structural Member

TRANSFORMATION OF LOAD DISTRIBUTIONS INTO STRESS DISTRIBUTIONS

The development of a practicable methodology for transforming distributions of loads to corresponding stress distributions required certain basic assumptions:

1. The influence of differential stresses resulting from the same gross load but different vehicular axle spacings (i. e., the same gross load but different equivalent "rectangular" load) is negligible (see Figures 7 through 10). If significant stress differentials are observed, some simple parameter (such as number of axles or total vehicle length) should be used to resolve these differences. The methodology employed here compromises these extremes. Instead of combining all vehicles into a single classification, three vehicle classes (autos, single-unit trucks, and combination trucks) were

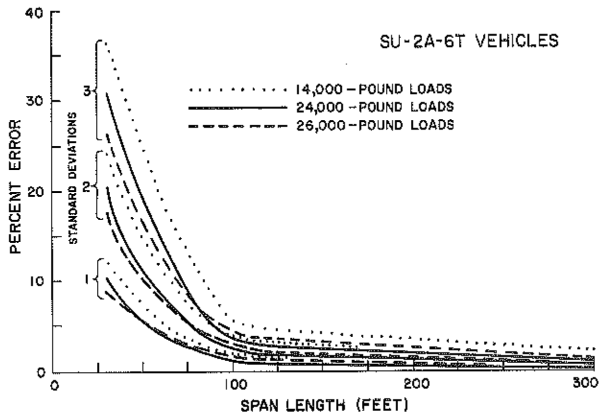

Figure 9. Errors in Computations for Single-Unit, Two-Axle, Six-Tire Vehicles

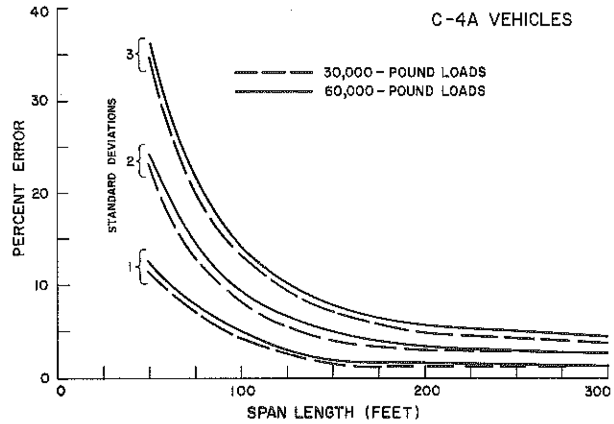

Figure 10. Errors in Computations for Combination, Four-Axle Vehicles

includes the variable of time, no problems will arise since these weight changes can be considered.

Load-Stress Relationship

Based on the above assumptions, generalized equations can be developed relating stress to the loading conditions. Immediately after erection of the bridge, the actual designed stress of a particular bridge member can be found from

$$S_d = (LL \times I + DL)/Z \quad 17$$

where Z is the cross-sectional area of the structural member.

Assuming that the percentage of section lost due to corrosion of a member is some function of time ($f_r(y)$), Equation 17 can be modified such that the design stress for a particular year can be computed from

$$S_d(y) = [LL(y) \times I + DL(y)] / Z\ (1 - \sum_{1}^{y} f_r(y)). \quad 18$$

It is seen that the term "designed" stress, S_d, is the stress in the member due to a specified design loading.

When rusting or values of impact vary over the years, the above relationship is subsequently modified. The impact factor depends not only on the geometrics of the bridge structure but also on the average running speed of the vehicles and the surface condition of the bridge deck and approaches. Figure 11 shows typical results when the above variables are duly considered. If the impact factor relationship I(y) can be quantitized, Equation 17 is further modified so that

$$S_d(y) = [LL(y) \times I(y) + DL(y)] / Z\ (1 - \sum_{1}^{y} f_r(y)). \quad 19$$

Load-Stress Curve

Assuming a linear relationship, points on the load-stress curve can be obtained as follows:

1. The origin of the load-stress axis (zero stress, zero load) (see Figure 12).
2. Stress due to dead load:

$$S_{DL}(y) = DL/Z(1 - \sum_{\text{all } y} f_r(y)) \quad 20$$

3. Maximum single load that can be carried by the member before yielding will occur:

$$LC(y) = Z\ (1 - \sum_{\text{all } y} f_r(y)) \times S(y) \times I(y). \quad 21$$

4. Minimum fatigue-producing load:

$$LC_{EL} = Z\ (1 - \sum_{\text{all } y} f_r(y)) \times f_{EL}(y) \times I(y) \quad 22$$

where f_{EL} corresponds to the endurance limit of the steel.

Cumulative Stress Distributions

Development of stress distributions (S_{TL_q}) from the load distributions (P_{TL_q}) is done by simply multiplying the frequencies of each loading interval in endurance limit,

4. A finite number of stress repetitions (N_{EL}) are required at the endurance limit before the member will fail, and
5. The slope of the S-log N curve between N_1 (at f_u) and N_{EL} (at f_{EL}) is constant; the slope of the S-log N curve between N_{EL} and N_∞ is zero.

Although the accuracies of the above assumptions are believed to produce fatigue histories well within the range of acceptable predictions, certain comments concerning some of these assumptions appear to be in order. The strength values (f_u and f_y) to be used as input into the analysis can be obtained by either extended laboratory tests or from information available from the manufacturer. If the latter source is utilized, the values of these parameters will probably be conservative.

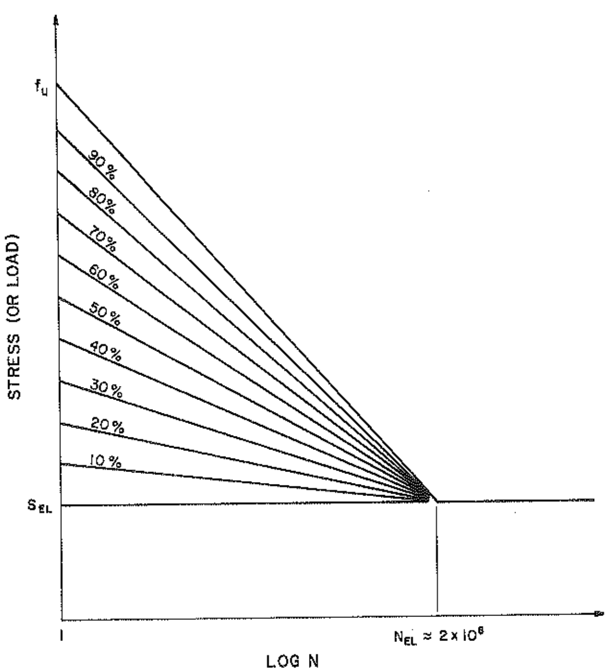

Figure 13. **Idealized Fatigue Curve**

A stress level of one-half the yield stress can be assumed for the endurance limit when information to the contrary is not available since this value is believed to be consistent with many steels presently used in bridge construction. The equations, developed later in this section, containing the parameter "endurance limit", however, consider this parameter as a variable.

The applicability of the assumption concerning the linearity of the S-log N curve is dependent on the type of material used. Most steels presently used in bridge construction have relationships approaching linearity. If this assumption cannot be considered applicable, the fatigue-stress relationships presented in the equations derived later in this section should be modified.

Fatigue Factors

Consider the typical S-log N curve presented in Figure 13. The slope (m) of this curve in the fatigue range (N_1 to N_{EL}) is found to be

$$m = -(f_u - f_{EL})/\log N_{EL} \; . \qquad 24$$

The generalized S-log N curve equation can then be obtained by substituting the above parameters into the generalized form of a linear equation so that

$$S_i = -[(f_u - f_{EL} \log N_i/\log N_{EL}] + f_u \; , \qquad 25$$

where N_i is the number of repetitions at the S_i stress level causing fatigue. Rearranging Equation 25 so that the dependent variable is in terms of the number of stress repetitions, the S-log N relationship becomes

$$\log N_i = (f_u - S_i) \log N_{EL}/ (f_u - f_{EL}) \; . \qquad 26$$

Comparing the N_i values to a base value of N_{EL} allows the computation of equivalent fatigue factors corresponding to differential stress levels. Designating this equivalency factor as the equivalent bridge loading (EBL), the equivalent number of endurance limit stressings required to fatigue a member to the same extent as one repetition of a S_i stress is found from

$$EBL_i = N_{EL} \times 10^{\log N_{EL} (S_i - f_u) / (f_u - f_{EL})} \qquad 27$$

If some base other than the endurance limit is desired, Equation 27 must be altered by developing a generalized relationship between the number of repetitions required for failure by fatigue and different stress interval by the corresponding EBL factor.

6. Sum the EBL's over all stress groups and compare the total to the maximum safe value -- N_{EL}.

Formulating Steps 5 and 6 as an equation, the percent of fatigue life (PFL) used during a design period of Y years is found from

$$PFL = 100 \sum_{\text{all } i} \sum_{\text{all } y} N_{iy} EBL_{iy} / N_{EL}, \qquad 35$$

where N_{iy} is the number of stress repetitions of the ith stress level using the bridge during the yth year and EBL_{iy} is the corresponding fatigue equivalency factor.

EXTENSION OF FATIGUE ANALYSIS METHODOLOGY FOR PREDICTIVE PURPOSES

The methodology for analyzing the fatigue condition of highway bridge members presented is amenable to all design periods, whether this period is from the erection date of the bridge to the present time or for some period into the future. Only the traffic data are of consequence with respect to the design period. If the period of time to be considèred is from the erection date to the present time, the question may be how much longer will the bridge withstand fatigue based on the available traffic data? For example, assume that the fatigue analysis indicated that 80 percent of the fatigue life of a particular bridge member has been reached. How many years from now will the member be expected to fail? How many years from now will the bridge member be fatigued to 95 percent of its life? These questions can be complicated by asking how many years of fatigue life will result if various load levels and (or) vehicle type restrictions are adopted? In order to answer such questions, certain assumptions concerning future traffic trends must be made.

PREDICTION OF FUTURE TRAFFIC FROM PAST TRENDS

Most simply, past traffic trends may be assumed to be indicative of future traffic characteristics. Because the various loading distributions from past traffic studies for a bridge are necessarily discrete, the extension of these parameters in the future is unreasonably tedious. Instead, it is recommended that a new traffic parameter be developed -- average EBL per vehicle (AEBL). This value is obtained for each time interval by dividing the total number of EBL's by the total number of vehicles. This ratio can then be plotted as a function of year to obtain AEBL over the design period.

The remaining parameter necessary for the development of the fatigue analysis is the ADT curve as a function of time. The portion of the curve representing the time from the bridge erection date to the time of the analysis is available from past traffic data.

Once the curves representing these parameters have been plotted, they are extrapolated into future years. The expected EBL's accumulated in any particular year is then found from

$$EBL(y) = 365 \times ADT(y) \times AEBL(y) \qquad 36$$

The total number of EBL's accumulated from the present time to the end of year Y can be computed from

$$TEBL = \sum_{\text{all } y} EBL(y)$$

PREDICTION OF FATIGUE LIFE WHEN TRAFFIC IS RESTRICTED

The effect of certain traffic restrictions might be considered. If the remaining life of a bridge is found to be four years, for example, the prospect of a bridge failing in four years might influence the engineer to try to extend its life by restricting certain vehicles from using the bridge or by lowering the posted maximum allowable gross load for the bridge. Such restrictions would result in a significant decrease in the accumulations of average EBL's per vehicle. Little effect would be realized in the average daily traffic (ADT) versus year relationship because few vehicles would be affected by the imposed restrictions. In fact, a slight increase in the ADT might be realized since the heavier vehicles may be replaced by a larger number of smaller vehicles.

Recomputation of the fatigue life would be similar to that presented already. This is accomplished by first

APPENDIX A

LIST OF SYMBOLS

MAXIMUM YIELD STRESS = 95000 PSI
ULTIMATE STRESS = 150000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
45000	49999	1.00
50000	54999	2.03
55000	59999	4.12
60000	64999	8.36
65000	69999	16.96
70000	74999	34.42
75000	79999	69.86
80000	84999	141.77
85000	89999	287.70
90000	94999	583.86
95000	99999	1184.88
100000	104999	2404.60
105000	104999	4879.88
110000	114999	9903.23
115000	119999	20097.59
120000	124999	40785.98
125000	129999	82771.00
130000	134999	167975.31
135000	139999	340888.37
140000	144999	691798.06
145000	149999	2000000.00

MAXIMUM YIELD STRESS = 65000 PSI
ULTIMATE STRESS = 125000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
30000	34999	1.00
35000	39999	2.19
40000	44999	4.80
45000	49999	10.51
50000	54999	23.03
55000	59999	50.46
60000	64999	110.56
65000	69999	242.20
70000	74999	530.60
75000	79999	1162.43
80000	84999	2546.62
85000	89999	5579.04
90000	94999	12222.38
95000	99999	26776.38
100000	104999	58660.78
105000	109999	128512.12
110000	114999	281539.81
115000	119999	616787.81
120000	124999	2000000.00

LIST OF SYMBOLS

ADT	Average Daily Traffic
AEBL	Average Equivalent Bridge Loading factor per vehicle
B	Constant
BL	Stress Base Level
D	Distance within which effects of vehicle placement are equal
DL	Dead Load
EBL	Equivalent Bridge Loading factor
F	Fatigue damage
f_{EL}	Stress at the Endurance Limit
$f_r(y)$	Rusting function
f_u	Ultimate stress or strength
f_y	Yield stress or strength
$f(F_{LC})$	Function relating the Fatigue to a stress level induced by the Loading Condition
f(LC)	Vehicle loading distribution
f(LL+WL+TC)	Loading function due to Live Load, Windload, and Temperature Changes
$f(S_{LC})$	Function relating the total loading to a stress level
GL	Gross Load interval
G_{mix}	Average Gap for Mixed traffic
G(t)	Average Gap of Time t
I	Impact factor
I(y)	Impact factor function
i	Vehicle type
j	Vehicle number
K_1 and K_2	Constants
L	Critical Length of roadway near midpoint of span
LC	Loading Combination

LL	Live Load
LC_E	Equivalent uniform Loading Condition
MN_i	Maximum Number of vehicles of type i
m	Slope of S–N curve
N	Number of repetitions of a stress
N_{EL}	Number of repetitions associated with the Endurance Limit
N_{n_1,n_2,n_3}	Number of repetitions of mixed loading configuration
n	Number of vehicles
n_i	Number of vehicles of type i
$\bar{P}$	Traffic composition Probability for two-lane, two-directional traffic
PFL	Percent of Fatigue Life
PR	Traffic composition Probability for r lanes of a one-directional highway
P_D	Probability that a vehicle is within length D
$P_{GL_{ijq}}$	Probability of the jth vehicle of type i being in Gross Load interval q
P_G	Probability of Gap length of G_{mix}
$P_{G(t)}$	Probability of Gap length of time t
P_{LC}	Probability of Loading Combination LC
P_{Li}	Gross vehicle Load Probability Distribution
P_{MN_i}	Probability of loading condition with MN_i vehicles of type i with gap length of $G(MN)_i$
P_{n_1,n_2,n_3}	Probability of mixed traffic loading
$P_{n_1,n_2,n_3,r}$	Probability of mixed traffic loading in lane r
P_{TL_q}	Total gross Load Probability distribution
P_i	Portion of total traffic being of vehicle type i
P_{ni}	Probability of n consecutive vehicles being of type i
P_{niG}	Probability of n consecutive vehicles being of type i with a Gap of G(t)
Q	Gross load or stress level
q	Number of gross load or stress intervals
r	Traffic lane

R_{TL_q}	Stress repetitions probability
S	Stress
SI	Stress Interval
SP	Average spot Speed
S_{DL}	Dead Load Stress
S_{TL_q}	Stress probability distribution
S_d	Designed Stress
S_i	Stress level
t	Time interval
TC	Load due to Temperature Change
TEBL	Total accumulated Equivalent Bridge Loading factor
TOT	Total number of vehicle groupings
VL	Average Vehicle Length
WL	Wind Load
W_{iq}	Mean of the qth load interval for vehicle type i
Y	Design Period
y	Year
Z	Sectional area of a structural member

APPENDIX B

EBL FACTORS

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C     PROGRAM FOR COMPUTING EBL FACTORS

      INTEGER YF,LOBD(60),UPBD(60)
      REAL MAXREP,LGMREP,FY,INT,DESIGN,BASE,INTS,MP(60),L,FACTOR(60),ULT
      MAXREP=2000000.
      LGMREP=ALOG10(MAXREP)
      READ(5,100)M
100   FORMAT(2X,I3)
      DO 1 K=1,M
      READ(5,200)FY,INT,ULT
200   FORMAT(2X,3F10.0)
      DESIGN=.50*FY
      BASE=DESIGN-.5*INT
      INTS=(ULT-DESIGN)/INT+1.4
      I=INTS
      DO 5 J=1,I
      MP(J)=O.
      FACTOR(J)=O.
      LOBD(J)=O
      UPBD(J)=O
  5   CONTINUE
      YF=FY
      IULT=ULT
      IBASE=BASE
      IINT=INT
      DO 2 J=1,I
      L=J
      MP(J)=DESIGN+(L-1.)*INT
      N=J
      LOBD(J)=IBASE+(N-1)*IINT
      UPBD(J)=IBASE+N*IINT-1
      IF(N.EQ.I) GO TO 3
      FACTOR(J)=MAXREP/(10**(LGMREP*(ULT-MP(J))/(ULT-.5*FY)))
      GO TO 2
  3   FACTOR(J)=MAXREP
  2   CONTINUE
      WRITE(6,300) YF,IINT,IULT
300   FORMAT(1H1,//13X,'MAXIMUM YIELD STRESS=',I6,' PSI',//16X,'STRESS
     1 INTERVAL=', I5,' PSI',//16X,'ULTIMATE STRESS=',I7,' PSI',///)
      WRITE(6,400)
400   FORMAT(10X,'STRESS INTERVAL(PSI)',2X,'EBL FACTOR',//)
      DO 4 J=1,I
      WRITE(6,500) LOBD(J),UPBD(J),FACTOR(J)
500   FORMAT(12X,2I7,5X,F11.2)
  4   CONTINUE
  1   CONTINUE
      WRITE(6,600)
600   FORMAT(1H1)
      CALL EXIT
      END
```

MAXIMUM YIELD STRESS = 25000 PSI
ULTIMATE STRESS = 48000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
12000	12999	1.00
13000	13999	1.50
14000	14999	2.26
15000	15999	3.41
16000	16999	5.13
17000	17999	7.72
18000	18999	11.61
19000	19999	17.48
20000	20999	26.30
21000	21999	39.58
22000	22999	59.56
23000	23999	89.63
24000	24999	134.87
25000	25999	202.97
26000	26999	305.43
27000	27999	459.63
28000	28999	691.68
29000	29999	1040.87
30000	30999	1566.36
31000	31999	2357.14
32000	32999	3547.13
33000	33999	5337.93
34000	34999	8032.77
35000	35999	12088.12
36000	36999	18190.85
37000	37999	27374.48
38000	38999	41194.56
39000	39999	61991.77
40000	40999	93288.44
41000	41999	140385.06
42000	42999	211258.69
43000	43999	317913.00
44000	44999	478411.87
45000	45999	719938.94
46000	46999	1083401.00
47000	47999	2000000.00

MAXIMUM YIELD STRESS = 33000 PSI
ULTIMATE STRESS = 39600 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
16000	16999	1.00
17000	17999	1.87
18000	18999	3.51
19000	19999	6.58
20000	20999	12.33
21000	21999	23.11
22000	22999	43.31
23000	23999	81.17
24000	24999	152.12
25000	25999	285.07
26000	26999	534.22
27000	27999	1001.14
28000	28999	1876.14
29000	29999	3515.89
30000	30999	6588.81
31000	31999	12347.50
32000	32999	23139.33
33000	33999	43363.33
34000	34999	81263.25
35000	35999	152288.19
36000	36999	285389.31
37000	37999	534822.37
38000	38999	1002261.44
39000	39999	2000000.00

MAXIMUM YIELD STRESS = 33000 PSI
ULTIMATE STRESS = 48000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
16000	16999	1.00
17000	17999	1.59
18000	18999	2.51
19000	19999	3.98
20000	20999	6.31
21000	21999	10.00
22000	22999	15.86
23000	23999	25.13
24000	24999	39.83
25000	25999	63.14
26000	26999	100.08
27000	27999	158.62
28000	28999	251.42
29000	29999	398.50
30000	30999	631.63
31000	31999	1001.14
32000	32999	1586.81
33000	33999	2515.12
34000	34999	3986.49
35000	35999	6318.62
36000	36999	10015.11
37000	37999	15874.05
38000	38999	25160.54
39000	39999	39879.80
40000	40999	63209.94
41000	41999	100188.62
42000	42999	158800.00
43000	43999	251700.06
44000	44999	398947.75
45000	45999	632337.19
46000	46999	1002261.44
47000	47999	2000000.00

MAXIMUM YIELD STRESS = 33000 PSI
ULTIMATE STRESS = 60000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
16000	16999	1.00
17000	17999	1.40
18000	18999	1.95
19000	19999	2.72
20000	20999	3.80
21000	21999	5.30
22000	22999	7.40
23000	23999	10.33
24000	24999	14.41
25000	25999	20.12
26000	26999	28.09
27000	27999	39.21
28000	28999	54.73
29000	29999	76.40
30000	30999	106.64
31000	31999	148.86
32000	32999	207.79
33000	33999	290.05
34000	34999	404.88
35000	35999	565.17
36000	36999	788.91
37000	37999	1101.23
38000	38999	1537.20
39000	39999	2145.76
40000	40999	2995.24
41000	41999	4181.04
42000	42999	5836.26
43000	43999	8146.79
44000	44999	11372.01
45000	45999	15874.09
46000	46999	22158.45
47000	47999	30930.73
48000	48999	43175.94
49000	49999	60268.80
50000	50999	84128.69
51000	51999	117434.19
52000	52999	163925.44
53000	53999	228821.69
54000	54999	319410.00
55000	55999	445861.31
56000	56999	622373.37
57000	57999	868764.31
58000	58999	1212699.00
59000	59999	2000000.00

MAXIMUM YIELD STRESS = 36000 PSI
ULTIMATE STRESS = 58000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
17500	18499	1.00
18500	19499	1.44
19500	20499	2.07
20500	21499	2.97
21500	22499	4.27
22500	23499	6.13
23500	24499	8.81
24500	25499	12.67
25500	26499	18.21
26500	27499	26.17
27500	28499	37.61
28500	29499	54.05
29500	30499	77.68
30500	31499	111.64
31500	32499	160.46
32500	34499	230.61
33500	34499	331.45
34500	35499	476.36
35500	36499	684.65
36500	37499	983.99
37500	38499	1414.22
38500	39499	2032.55
39500	40499	2921.25
40500	41499	4198.50
41500	42499	6034.20
42500	43499	8672.52
43500	44499	12464.42
44500	45499	17914.19
45500	46499	25746.79
46500	47499	37003.95
47500	48499	53183.04
48500	49499	76436.12
49500	50499	109856.37
50500	51499	157888.38
51500	52499	226921.50
52500	53499	326138.12
53500	54499	468735.00
54500	55499	673679.44
55500	56499	968230.25
56500	57499	1391567.00
57500	58499	2000000.00

MAXIMUM YIELD STRESS = 46000 PSI
ULITMATE STRESS = 67000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
22500	23499	1.00
23500	24499	1.39
24500	25499	1.93
25500	26499	2.69
26500	27499	3.74
27500	28499	5.20
28500	29499	7.23
29500	30499	10.06
30500	31499	13.98
31500	32499	19.45
32500	33499	27.04
33500	34499	37.61
34500	35499	52.30
35500	36499	72.72
36500	37499	101.13
37500	38499	140.63
38500	39499	195.56
39500	40499	271.95
40500	41499	378.18
41500	42499	525.90
42500	43499	731.32
43500	44499	1016.98
44500	45499	1414.22
45500	46499	1966.63
46500	47499	2734.81
47500	48499	3803.05
48500	49499	5288.56
49500	50499	7354.32
50500	51499	10226.99
51500	52499	14221.74
52500	53499	19776.95
53500	54499	27501.96
54500	55499	38244.54
55500	56499	53183.15
56500	57499	73956.87
57500	58499	102845.00
58500	59499	143017.44
59500	60499	198881.69
60500	61499	276566.06
61500	62499	384595.50
62500	63499	534822.37
63500	64499	743728.37
64500	65499	1034235.75
65500	66499	1438218.00
66500	67499	2000000.00

MAXIMUM YIELD STRESS = 20000 PSI
ULTIMATE STRESS = 30000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
9750	10249	1.00
10250	10749	1.44
10750	11249	2.07
11250	11749	2.97
11750	12249	4.27
12250	12749	6.13
12750	13249	8.81
13250	13749	12.67
13750	14249	18.21
14250	14749	26.17
14750	15249	37.61
15250	15749	54.05
15750	16249	77.68
16250	16749	111.64
16750	17249	160.46
17250	17749	230.61
17750	18249	331.45
18250	18749	476.37
18750	19249	684.65
19250	19749	983.99
19750	20249	1414.22
20250	20749	2032.55
20750	21249	2921.24
21250	21749	4198.49
21750	22249	6034.20
22250	22749	8672.52
22750	23249	12464.36
23250	23749	17914.14
23750	24249	25746.74
24250	24749	37003.95
24750	25249	53183.04
25250	25749	76436.12
25750	26249	109856.37
26250	26749	157888.38
26750	27249	226921.50
27250	27749	326138.12
27750	28249	468735.00
28250	28749	673679.44
28750	29249	968230.25
29250	29747	1391567.00
29750	30249	2000000.00

MAXIMUM YIELD STRESS = 40000 PSI
ULTIMATE STRESS = 60000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
19500	20499	1.00
20500	21499	1.44
21500	22499	2.07
22500	23499	2.97
23500	24499	4.27
24500	25499	6.13
25500	26499	8.81
26500	27499	12.67
27500	28499	18.21
28500	29499	26.17
29500	30499	37.61
30500	31499	54.05
31500	32499	77.68
32500	33499	111.64
33500	33499	160.46
34500	35499	230.61
35500	36499	331.45
36500	37499	476.36
37500	38499	684.65
38500	39499	983.99
39500	40499	1414.22
40500	41499	2032.55
41500	42499	2921.25
42500	43499	4198.50
43500	44499	6034.20
44500	45499	8672.52
45500	46499	12464.42
46500	47499	17914.19
47500	48499	25746.79
48500	49499	37003.95
49500	50499	53183.04
50500	51499	76436.12
51500	52499	109856.37
52500	53499	157888.38
53500	54499	226921.50
54500	55499	326138.12
55500	54499	468735.00
56500	57499	673679.44
57500	58499	968230.25
58500	59499	1391567.00
59500	60499	2000000.00

MAXIMUM YIELD STRESS = 50000 PSI
ULTIMATE STRESS = 70000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
24000	25999	1.00
26000	27999	1.91
28000	29999	3.63
30000	31999	6.92
32000	33999	13.19
34000	35999	25.13
36000	37999	47.89
38000	39999	91.27
40000	41999	173.93
42000	43999	331.45
44000	45999	631.62
46000	47999	1203.66
48000	49999	2293.77
50000	51999	4371.16
52000	53999	8329.95
54000	55999	15874.05
56000	57999	30250.60
58000	59999	57647.39
60000	61999	109856.37
62000	63999	209348.56
64000	65999	398947.75
66000	67999	760258.81
68000	69999	2000000.00

MAXIMUM YIELD STRESS = 50000 PSI
ULTIMATE STRESS = 70000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
24500	25499	1.00
25500	26499	1.38
26500	27499	1.91
27500	28499	2.63
28500	29499	3.63
29500	30499	5.01
30500	31499	6.92
31500	32499	9.55
32500	33499	13.19
33500	34499	18.21
34500	35499	25.13
35500	36499	34.69
36500	37499	47.89
37500	38499	66.12
38500	39499	91.27
39500	40499	125.99
40500	41499	173.93
41500	42499	240.10
42500	43499	331.45
43500	44499	457.55
44500	45499	631.62
45500	46499	871.93
46500	47499	1203.66
47500	48499	1661.61
48500	49499	2293.77
49500	50499	3166.46
50500	51499	4371.16
51500	52499	6034.20
52500	53499	8329.95
53500	54499	11499.15
54500	55499	15874.05
55500	56499	21913.52
56500	57499	30250.60
57500	58499	41759.70
58500	59499	57647.39
59500	60499	79579.63
60500	61499	109856.37
61500	62499	151651.63
62500	63499	209348.56
63500	64499	288996.94
64500	65499	398947.75
65500	66499	550730.06
66500	67499	760258.81
67500	68499	1049504.00
68500	69499	1448795.00
69500	70499	2000000.00

MAXIMUM YIELD STRESS = 42000 PSI
ULTIMATE STRESS = 63000 PSI

STRESS INTERVAL(PSI)		EBL FACTOR
20500	21499	1.00
21500	22499	1.41
22500	23499	2.00
23500	24499	2.82
24500	25499	3.98
25500	26499	5.63
26500	27499	7.95
27500	28499	11.22
28500	29499	15.86
29500	30499	22.40
30500	31499	31.64
31500	32499	44.70
32500	33499	63.14
33500	34499	89.19
34500	35499	125.99
35500	36499	177.98
36500	37499	251.42
37500	38499	355.16
38500	39499	501.70
39500	40499	708.71
40500	41499	1001.14
41500	42499	1414.22
42500	43499	1997.75
43500	44499	2822.06
44500	45499	3986.49
45500	46499	5631.37
46500	47499	7954.99
47500	48499	11237.34
48500	49499	15874.09
49500	50499	22423.96
50500	51499	31676.52
51500	52499	44746.78
52500	53499	63209.94
53500	54499	89291.37
54500	55499	126134.81
55500	56499	178180.19
56500	57499	251700.06
57500	58499	355555.62
58500	59499	502264.31
59500	60499	709506.87
60500	61499	1002261.44
61500	62499	1415812.00
62500	63499	2000000.00